Problem 7.49

A force $\vec{F} = (2x + 4)\hat{i}$ acts on a 5.00 kg mass. As the particle moves between x = 1.00 meters and x = 5.00 meters:

a.) How much work does "F" do during the motion?

$$\begin{aligned} W &= \int dW \\ &= \int \vec{F} \bullet d\vec{r} \\ &= \int_{x=1.00}^{x=5.00} \left((2x+4)\hat{i}\right) \bullet \left((dx)\hat{i}\right) \\ &= \int_{x=1.00}^{x=5.00} (2x+4)\,dx \\ &= \left(x^2 + 4x\right)\Big|_{x=1.00}^{x=5.00} \\ &= \left((5.00\text{ m})^2 + 4(5.00\text{ m})\right) - \left((1.00\text{ m})^2 + 4(1.00\text{ m})\right) \\ &= 40.0\text{ J} \end{aligned}$$

b.) How much change of potential energy must occur between the points?

This question is designed to make you think about the relationship between *work done by a conservative force* and *potential energy*. In fact, the relationship is:

$$W_{consForce} = -\Delta U$$

In *Part a*, we calculated the amount of work done by the conservative force as 40.0 joules. Apparently, according to our relationship, that means:

$$\Delta U = -40.0\text{ J}$$

c.) The speed the particle ends up with after the run, assuming that at the beginning its speed was 3.00 m/s?

Dang! I was so hoping for a *conservation of energy* problem. Unfortunately, this is just another Work/Energy problem.

c.) The speed the particle ends up with after the run, assuming that at the beginning its speed was 3.00 m/s?

$$\begin{aligned} &W_{consForce} = -\Delta U \\ &W_{net} = \Delta KE \\ &W_{net} = W_{consForce} \\ &\Rightarrow \quad -\Delta U = \Delta KE \\ &\Rightarrow \quad -(-40.0\text{ J}) = KE_2 - \frac{1}{2}mv_1^2 \\ &\Rightarrow \quad KE_2 = (40.0\text{ J}) + \frac{1}{2}mv_1 \\ &\qquad = (40.0\text{ J}) + \frac{1}{2}(5.00\text{ kg})(3.00\text{ m/s})^2 \\ &\qquad = 62.5\text{ J} \end{aligned}$$